Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Wednesday 22 May 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/01**

Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 7 5 6 8 2 A 0 1 3 2

Pearson

1. $$f(z) = z^4 - 6z^3 + az^2 + bz + 145$$

where a and b are real constants.

Given that $2 + 5i$ is a root of the equation $f(z) = 0$

(a) determine the other roots of the equation $f(z) = 0$

(7)

(b) Show all the roots of $f(z) = 0$ on a single Argand diagram.

(2)

1

a) if $2+5i$ is a root, $2-5i$ is also a root

let $\alpha = 2+5i$ $\quad \alpha+\beta = 4 \quad \Rightarrow z^2 - 4z + 29 = 0$ 1

$\beta = 2-5i \quad \alpha\beta = 29$ 1

1

$\therefore f(z) = (z^2 - 4z + 29)(z^2 + cz + d) = z^4 - 6z^3 + az^2 + bz + 145$

comparing coefficients: constant: $29d = 145 \Rightarrow d = 5$

z^3: $c - 4 = -6$

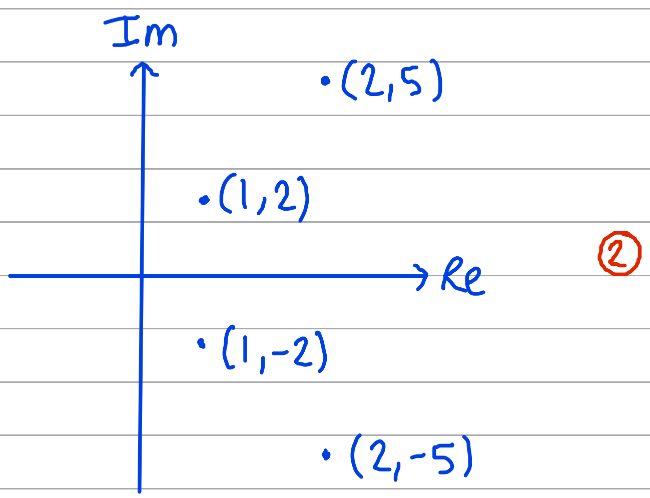
$c = -2$

$f(z) = (z^2 - 4z + 29)(z^2 - 2z + 5)$ 1

1

if $z^2 - 2z + 5 = 0$, $z = 1 \pm 2i$ 1

b)



2

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 9 marks)

2. The roots of the equation

$$2x^3 - 3x^2 + 12x + 7 = 0$$

are α, β and γ

Without solving the equation,

(a) write down the value of each of

$$\alpha + \beta + \gamma \qquad \alpha\beta + \alpha\gamma + \beta\gamma \qquad \alpha\beta\gamma$$

(1)

(b) Use the answers to part (a) to determine the value of

(i) $\dfrac{2}{\alpha} + \dfrac{2}{\beta} + \dfrac{2}{\gamma}$

(ii) $(\alpha - 1)(\beta - 1)(\gamma - 1)$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

(7)

a) $\alpha + \beta + \gamma = \frac{3}{2}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{12}{2} = 6$, $\alpha\beta\gamma = -\frac{7}{2}$ 1

b) (i) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma} = \frac{2(\alpha\beta + \beta\gamma + \gamma\alpha)}{\alpha\beta\gamma} = \frac{2 \times 6}{-7/2}$ 1 $= -\frac{24}{7}$ 1

(ii) $(\alpha - 1)(\beta - 1)(\gamma - 1) = (\alpha\beta - \alpha - \beta + 1)(\gamma - 1)$ 1

$= \alpha\beta\gamma - \alpha\beta - \alpha\gamma + \alpha - \beta\gamma + \beta + \gamma - 1$

$= \alpha\beta\gamma - (\alpha\beta + \beta\gamma + \gamma\alpha) + (\alpha + \beta + \gamma) - 1$ 1

$= -\frac{7}{2} - 6 + \frac{3}{2} - 1 = -9$ 1

(iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= \left(\frac{3}{2}\right)^2 - 2(6)$ 1

$= -\frac{39}{4}$ 1

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

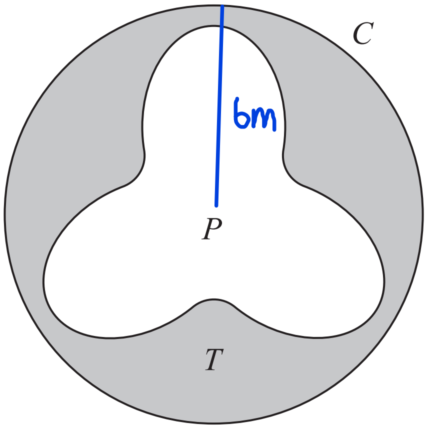


Figure 1

Figure 1 shows the design for a bathing pool.

The pool, P, shown unshaded in Figure 1, is surrounded by a tiled area, T, shown shaded in Figure 1.

The tiled area is bounded by the edge of the pool and by a circle, C, with radius 6 m.

The centre of the pool and the centre of the circle are the same point.

The edge of the pool is modelled by the curve with polar equation

$$r = 4 - a\sin 3\theta \qquad 0 \leqslant \theta \leqslant 2\pi$$

where a is a positive constant.

Given that the shortest distance between the edge of the pool and the circle C is 0.5 m,

(a) determine the value of a.

(2)

(b) Hence, using algebraic integration, determine, according to the model, the exact area of T.

(6)

a) max r is when $\sin 3\theta = -1$, $r = 4 + a$

1

$4 + a = 5.5 \Rightarrow a = 1.5$ 1

b) Area of pool $= \frac{1}{2}\int_0^{2\pi} (4 - 1.5\sin 3\theta)^2 \, d\theta$ 1

Question 3 continued

$$(4 - 1.5\sin 3\theta)^2 = 16 - 12\sin 3\theta + 2.25\sin^2 3\theta$$

$$= 16 - 12\sin 3\theta + 2.25\left(\frac{1-\cos 6\theta}{2}\right)$$

$$= \frac{137}{8} - 12\sin 3\theta - \frac{9}{8}\cos 6\theta \quad (1)$$

$$\int\left(\frac{137}{8} - 12\sin 3\theta - \frac{9}{8}\cos 6\theta\right)d\theta = \left[\frac{137\theta}{8} + 4\cos 3\theta - \frac{3}{16}\sin 6\theta\right] \quad (1)$$

$$\therefore \frac{1}{2}\int_0^{2\pi}(4-1.5\sin 3\theta)^2 = \frac{1}{2}\left[\frac{137\theta}{8} + 4\cos 3\theta - \frac{3}{16}\sin 6\theta\right]_0^{2\pi}$$

$$= \frac{137}{8}\pi \quad (1)$$

$$\text{Area of } T = \underbrace{\pi \times 6^2}_{\text{area } C} - \underbrace{\frac{137}{8}\pi}_{\text{area } P} \overset{(1)}{=} \frac{151}{8}\pi \text{ m}^2 \quad (1)$$

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4. The complex number $z = e^{i\theta}$, where θ is real.

(a) Show that

$$z^n + \frac{1}{z^n} \equiv 2\cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$$

(5)

(c) Hence, making your reasoning clear, determine all the solutions of

$$\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$$

in the interval $0 \leqslant \theta < 2\pi$

(3)

a) $z^n + \frac{1}{z^n} = e^{in\theta} + \frac{1}{e^{in\theta}} = e^{in\theta} + e^{-in\theta}$ 1

$= \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$

$= 2\cos n\theta$ 1

b) $\left(z + \frac{1}{z}\right)^5 = (2\cos n\theta)^5 = 32\cos^5 n\theta$ 1

using binomial expansion:

$\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z}\right)^2 + 10z^2\left(\frac{1}{z}\right)^3$

$+ 5z\left(\frac{1}{z}\right)^4 + \left(\frac{1}{z}\right)^5$

$= z^5 + 5z^3 + 10z + \frac{10}{z} + \frac{5}{z^3} + \frac{1}{z^5}$ 1 1

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Question 4 continued

$$= \left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$$

$$32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta \quad 1$$

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta) \text{ as required} \quad 1$$

c) $\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta$

$$16\cos^5\theta = -2\cos\theta \quad 1$$

$$2\cos\theta(8\cos^4\theta + 1) = 0 \quad 1$$

$$\cos^4\theta = -\frac{1}{8} \text{ no solutions}$$

$$\therefore \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad 1$$

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

5. A raindrop falls from rest from a cloud. The velocity, v m s^{-1} vertically downwards, of the raindrop, t seconds after the raindrop starts to fall, is modelled by the differential equation

$$(t+2)\frac{dv}{dt} + 3v = k(t+2) - 3 \qquad t \geqslant 0$$

where k is a positive constant.

(a) Solve the differential equation to show that

$$v = \frac{k}{4}(t+2) - 1 + \frac{4(2-k)}{(t+2)^3}$$

(5)

Given that $v = 4$ when $t = 2$

(b) determine, according to the model, the velocity of the raindrop 5 seconds after it starts to fall.

(3)

(c) Comment on the validity of the model for very large values of t

(1)

a) $\frac{dv}{dt} + \frac{3v}{t+2} = k - \frac{3}{t+2}$

Integrating factor $= e^{\int \frac{3}{t+2} dt} = e^{3\ln(t+2)} = (t+2)^3$ 1

$(t+2)^3 \frac{dv}{dt} + 3v(t+2)^3 = k(t+2)^3 - 3(t+2)^2$

$\frac{d}{dt}(v(t+2)^3) = k(t+2)^3 - 3(t+2)^2$

$v(t+2)^3 = \int (k(t+2)^3 - 3(t+2)^2) dt$ 1

$v(t+2)^3 = \frac{k}{4}(t+2)^4 - (t+2)^3 + c$ 1

$v = \frac{k}{4}(t+2) - 1 + c(t+2)^{-3}$

sub in $t=0, v=0$:

$0 = \frac{k}{4}(0+2) - 1 + c(0+2)^{-3}$

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Question 5 continued

$0 = \frac{1}{2}k - 1 + \frac{c}{8} \Rightarrow c = 8 - 4k$ (1)

$v = \frac{k}{4}(t+2) - 1 + (8-4k)(t+2)^{-3}$

$v = \frac{k}{4}(t+2) - 1 + \frac{4(2-k)}{(t+2)^3}$ as required (1)

b) sub in $v=4$, $t=2$:

$4 = \frac{k}{4}(2+2) - 1 + \frac{4(2-k)}{(2+2)^3}$

$4 = k - 1 + \frac{8-4k}{64}$

$4 = k - 1 + \frac{1}{8} - \frac{1}{16}k$

$k = 5.2$ (1)

when $t=5$, $v = \frac{5.2}{4}(5+2) - 1 + 4(2-5.2)(5+2)^{-3}$ (1)

$= 8.06\,ms^{-1}$ (3sf) (1)

c) The model suggests that the speed increases indefinitely which is unlikely (1)

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n\left(4n^2-1\right)$$

(6)

Base case $n=1$: LHS: $\sum_{r=1}^{1}(2r-1)^2 = (2-1)^2 = 1$

RHS: $\frac{1}{3}(1)(4\times1^2-1) = 1$

LHS = RHS so true for $n=1$ 1

Assume true for $n=k$: $\sum_{r=1}^{k}(2r-1)^2 = \frac{1}{3}k(4k^2-1)$

Show true for $n=k+1$:

$$\sum_{r=1}^{k+1}(2r-1)^2 = \sum_{r=1}^{k}(2r-1)^2 + (2(k+1)-1)^2 \quad 1$$

$$= \frac{1}{3}k(4k^2-1) + (2k+1)^2$$

$$= \frac{1}{3}(2k+1)[k(2k-1)+3(2k+1)]$$

$$= \frac{1}{3}(2k+1)[2k^2-k+6k+3]$$

$$= \frac{1}{3}(2k+1)(2k^2+5k+3) \quad 1$$

$$= \frac{1}{3}(k+1)(2k+3)(2k+1) \quad 1$$

$$= \frac{1}{3}(k+1)(4(k+1)^2-1) \quad 1$$

so is true for $n=k+1$. If the statement is true for $n=k$ then it is true for $n=k+1$. Since it is true for $n=1$, it is true for all positive integers n. 1

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Question 6 continued

(Total for Question 6 is 6 marks)

7. The line l_1 has equation

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 4\mathbf{k})$$

and the line l_2 has equation

$$\mathbf{r} = 5\mathbf{i} + p\mathbf{j} - 7\mathbf{k} + \mu(6\mathbf{i} + \mathbf{j} + 8\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

The plane Π contains l_1 and l_2

(a) Show that the vector $3\mathbf{i} - 10\mathbf{j} - \mathbf{k}$ is perpendicular to Π

(2)

(b) Hence determine a Cartesian equation of Π

(2)

(c) Hence determine the value of p

(2)

Given that

- the lines l_1 and l_2 intersect at the point A
- the point B has coordinates $(12, -11, 6)$

(d) determine, to the nearest degree, the acute angle between AB and Π

(4)

a) $\begin{pmatrix} 3 \\ -10 \\ -1 \end{pmatrix}$ is perpendicular to Π if it is perpendicular to Π's two direction vectors.

$$\begin{pmatrix} 3 \\ -10 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} = 6 - 10 + 4 = 0 \checkmark$$

$$\begin{pmatrix} 3 \\ -10 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 1 \\ 8 \end{pmatrix} = 18 - 10 - 8 = 0 \checkmark \quad 1$$

so $\begin{pmatrix} 3 \\ -10 \\ -1 \end{pmatrix}$ is perpendicular to Π 1

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Question 7 continued

b) using $\underline{r}\cdot\underline{n} = \underline{a}\cdot\underline{n}$: $\underline{a}$ is a point on the plane

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -10 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -10 \\ -1 \end{pmatrix} = 3 + 20 - 3 = 20 \quad (1)$$

$$3x - 10y - z = 20 \quad (1)$$

c) $\begin{pmatrix} 5 \\ p \\ -7 \end{pmatrix}$ is in Π: $5(3) - 10p - (-7) = 20$ (1)

$$15 - 10p + 7 = 20$$
$$p = 0.2 \quad (1)$$

d) finding A: setting $l_1 = l_2$

$$\begin{pmatrix} 1+2\lambda \\ -2+\lambda \\ 3-4\lambda \end{pmatrix} = \begin{pmatrix} 5+6\mu \\ 0.2+\mu \\ -7+8\mu \end{pmatrix}$$

$1 + 2\lambda = 5 + 6\mu \Rightarrow 2\lambda - 6\mu = 4$ (1)
$-2 + \lambda = 0.2 + \mu \Rightarrow \lambda - \mu = 2.2$ (2)

solve (1) and (2) simultaneously using calculator: $\lambda = 2.3$, $\mu = 0.1$ (1)

check with third row: $3 - 4(2.3) = -7 + 8(0.1)$ ✓
$-6.2 = -6.2$

so $A = \begin{pmatrix} 5.6 \\ 0.3 \\ -6.2 \end{pmatrix}$

$$AB = \begin{pmatrix} 12 \\ -11 \\ 6 \end{pmatrix} - \begin{pmatrix} 5.6 \\ 0.3 \\ -6.2 \end{pmatrix} = \begin{pmatrix} 6.4 \\ -11.3 \\ 12.2 \end{pmatrix} \quad (1)$$

Question 7 continued

angle between AB and Π:

remember to use $\sin\theta$ for angles between a line and a plane

$$\sin\theta = \frac{\begin{pmatrix} 6.4 \\ -11.3 \\ 12.2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -10 \\ -1 \end{pmatrix}}{\sqrt{6.4^2 + 11.3^2 + 12.2^2}\sqrt{3^2 + 10^2 + 1^2}} \quad 1$$

$\theta = 40°$ (nearest degree) 1

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Question 7 continued

(Total for Question 7 is 10 marks)

8. A scientist is studying the effect of introducing a population of type A bacteria into a population of type B bacteria.

At time t days, the number of type A bacteria, x, and the number of type B bacteria, y, are modelled by the differential equations

$$\frac{dx}{dt} = x + y \quad (1)$$

$$\frac{dy}{dt} = 3y - 2x \quad (2)$$

(a) Show that

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 5x = 0$$

(3)

(b) Determine a general solution for the number of type A bacteria at time t days.

(4)

(c) Determine a general solution for the number of type B bacteria at time t days.

(2)

The model predicts that, at time T hours, the number of bacteria in the two populations will be equal.

Given that $x = 100$ and $y = 275$ when $t = 0$

(d) determine the value of T, giving your answer to 2 decimal places.

(5)

(e) Suggest a limitation of the model.

(1)

a) from (1): $y = \dot{x} - x \Rightarrow \dot{y} = \ddot{x} - \dot{x}$ (1)

sub into (2): $\ddot{x} - \dot{x} = 3(\dot{x} - x) - 2x$ (1)

$\ddot{x} - \dot{x} = 3\dot{x} - 3x - 2x$

$\ddot{x} - 4\dot{x} + 5x = 0$

$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 5x = 0$ as required (1)

(1) (1)

b) auxillary equation: $m^2 - 4m + 5 = 0 \Rightarrow m = 2 \pm i$

general solution: $x = e^{2t}(A\cos t + B\sin t)$

(1) (1)

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Question 8 continued

c) $y = \frac{dx}{dt} - x$ so need to find $\frac{dx}{dt}$

$$\frac{dx}{dt} = e^{2t}(B\cos t - A\sin t) + 2e^{2t}(A\cos t + B\sin t)$$

1

$$y = e^{2t}(B\cos t - A\sin t + 2A\cos t + 2B\sin t) + e^{2t}(A\cos t + B\sin t)$$

$$y = e^{2t}((A+B)\cos t + (B-A)\sin t) \quad 1$$

d) sub in $t=0$, $x=100$:

$$100 = e^0(A\cos 0 + B\sin 0)$$
$$A = 100$$

sub in $t=0$, $y=275$, $A=100$

$$275 = e^0((B+100)\cos 0 + (B-100)\sin 0)$$
$$275 = B + 100$$
$$B = 175 \quad 1$$

set $x=y$:

$$e^{2t}(100\cos t + 175\sin t) = e^{2t}(275\cos t + 75\sin t)$$

$$100\cos t + 175\sin t = 275\cos t + 75\sin t$$

$$100\sin t = 175\cos t \quad 1$$

$$\tan t = 1.75 \quad 1$$

$$t = 1.05...$$

1 1

But T is in hours so $T = 24 \times 1.05 = 25.24$ hours (2dp)

Question 8 continued

e) Both populations become negative at some points, which is impossible. 1

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Question 8 continued

Question 8 continued

(Total for Question 8 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS